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ABSTRACT

Pilot research using a draft version of the Mathematics Inventory is discussed in this report. The Mathematics Inventory is a self evaluation instrument which covers five major subject areas: elementary algebra, intermediate/advanced algebra, plane and solid geometry, trigonometry, and coordinate geometry and functions. The instrument is considered experimental and must be subjected to proper validation. The report concludes with several appendices. Included are the Mathematics Inventory, with instructions and answer sheet; a prototype report to the student based on his or her responses to the inventory; and a prototype report to the secondary school. (DWH)

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College Board Report

No. 84-6

A Profile of Preparation in Mathematics

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TM 850 114

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Educational Testing Service

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ABSTRACT

A self-evaluation instrument entitled the Mathematics Inventory was developed whose purpose is to generate a profile of a student's preparation in core areas of secondary school mathematics. This report discusses pilot research using a draft version of the inventory. Appended to the report are the inventory along with prototypes of reports to students and to schools that could be generated from the students' inventory responses.

INTRODUCTION

The Mathematics Inventory[1] covers five major subject areas: elementary algebra, intermediate/advanced algebra, plane and solid geometry, trigonometry, and coordinate geometry and functions. Within each area are listed a number of topics (for example, solution of quadratic equations), with one or more illustrative problems per topic. The students are to make a response for each topic that indicates how well they had learned the topic.

The current project has been conducted in two phases. This report deals with Phase II, in which a draft of the Mathematics Inventory was pilot tested with a sample of secondary school students, and prototypes of the reports to students and to schools were developed. Prior to discussion of Phase II, however, it is useful to present a brief overview of Phase I, in which this draft of the inventory was developed.

Phase I Activities

Six mathematics educators served as external consultants in Phase I, including three persons representing the secondary school perspective and three representing the college and university perspective. (See Appendix A for a list of consultants.) Through meetings with these consultants, key areas and topics in mathematics were identified from a larger list that had been compiled from secondary school texts, specifications for existing tests, and other sources. Illustrative problems were also constructed which, together with the list of subject areas and topics, formed the basic structure of a preliminary draft of the inventory.

Although efforts had been made to limit the length of the preliminary drafts, the consultants and project staff felt that the inventory still needed to be condensed in order to maximize its utility. Thus, input was obtained from a larger group of mathematics educators to determine which topics in the inventory were considered to be most important. These reviewers' ratings led to condensation of the inventory into the draft version submitted to pilot testing in Phase II.

Phase II Activities

In Phase II of the project, the following activities were conducted: (a) data from the draft inventory were collected from a sample of 363 high school students in the eleventh and twelfth grades in order to identify areas in which the inventory might be further improved; (b) portions of the student sample completed a questionnaire or were interviewed in order to ascertain their comments about the inventory and its purpose; and (c) consultants in mathematics were contacted by phone, and staff members at Educational Testing Service were consulted, to discuss possible improvements in the inventory and to discuss the nature of the reports to the students and the schools that could be prepared on the basis of the students' responses.

INVENTORY RESPONSE ANALYSIS

High school students were asked to complete the inventory, in order to address several issues. One set of issues concerned the feasibility of the instrument and the time required to complete it. Another set of issues concerned interrelationships among topics and items (sample problems) within topics. Preliminary data were also collected regarding the relationship of the inventory data to other measures such as Scholastic Aptitude Test (SAT) scores, self-reported mathematics grades, and actual mathematics grades.

It should be noted that this data collection activity involved a quite small sample; data analyses were necessarily conducted separately for eleventh and twelfth graders (and within grades, separate analyses were conducted for the different forms described below). So the samples on which the data analyses were conducted were too limited to permit conclusions about the validity of the inventory in relation to other measures. Nevertheless, this activity was expected to serve a useful function in refinement of the inventory. In particular, the examination of inter-item correlations was expected to indicate areas in which the inventory might be highly redundant and, thus, subject to further condensation. Also, assessment of correlations between the inventory responses and other measures could provide hypotheses about the inventory's concurrent validity that could be tested in larger-scale research.

1. The Mathematics Inventory presented in this report has undergone initial development and pilot testing. It remains to be subjected to validation, however, and thus is still an experimental instrument. Nevertheless, the inventory is being made available at this time for use by institutions, groups, and individuals. Interested parties are encouraged to collect data and conduct studies with the inventory. Such data collection efforts will help to establish the validity of the instrument and to demonstrate ways in which it can be most useful to mathematics educators and students.

Method

The sample

The total sample consisted of 363 students, drawn from seven different schools—one in New York City, one in Philadelphia, and five in urban, suburban, and rural areas of New Jersey. The participating schools are listed in Appendix B. Each school contributed between 13 and 18 percent of the sample. Two were private schools (one an all-boys' school) and five were public schools. The sample included 75 eleventh-grade boys, 80 eleventh-grade girls, 118 twelfth-grade boys, and 90 twelfth-grade girls. The students completed the inventory during regularly scheduled mathematics classes in April 1983. Classes were chosen to represent a range of levels of curriculum and student ability, with the restriction that substantial numbers of the students intend to enroll in college, since the Mathematics Inventory was developed primarily as an instrument to be used for college-bound students. (Of 60 students completing a questionnaire after the inventory, 55 indicated definite plans to attend a four-year college, while the others were either not sure or planned to attend a two-year college or other institution.)

Scores on the SAT were available for most students in the sample. Course grades for many of the students were also available from their school records. Grades in some schools were assigned on a letter scale and, in some schools, on a 100-point numerical scale. In order to translate all grades to the same scale, the following conversion table was used:

A (89–100) = 9; A− (85–88) = 8; B+ (82–84) = 7; B (79–81) = 6; B− (75–78) = 5; C+ (72–74) = 4; C (69–71) = 3; C− (65–68) = 2; D or below (64 or below) = 1.

Also available from the students' records were the averages of the students' six self-reported grades on the Student Descriptive Questionnaire (SDQ)—that is, the latest grades in English, mathematics, foreign language, biological sciences, physical sciences, and social sciences, expressed on a five-point scale with A = 4, and F = 0.

The average SAT-mathematical score of the students in the sample was 541 (S.D. = 97), and the average SAT-verbal score was 480 (S.D. = 91). Data collected in three recent national administrations with juniors and seniors showed mean SAT-mathematical scores ranging from 453 to 488 and SAT-verbal scores ranging from 404 to 431. (The College Entrance Examination Board, 1983-84. *ATP Guide for high schools and colleges: SAT and achievement tests.* New York: Author.) Thus, the present sample was higher than the above-mentioned national samples in average SAT-mathematical score and SAT-verbal score. For those students for whom course grades were available, the average grades (on the scale presented above) in first-year algebra, second-year algebra, and geometry, respectively, were 6.5, 6.1, and 5.7 (S.D.s = 2.5, 2.6, and 2.8). The average of the self-reported grades on the SDQ for the sample was 3.27 (S.D. = .51) on a scale from 0 to 4.

The instrument

Some of the students received the inventory in its regular format: A number of topics were presented in each of the five subject areas, with sample problems for each topic, and the students indicated, for each topic, how successful they had been at solving problems in that topic. This version was labeled Form 1. (The version of the inventory shown in Appendix E, although a later revision, is quite similar to the Form 1 inventory administered to the students.)[2]

A total of 30 eleventh graders and 42 twelfth graders received Form 1. Other students received modified versions of the inventory (labeled Forms 2A and 2B), in which they responded to each individual problem, indicating how successful they had been at solving that type of problem. Forms 2A and 2B were used to obtain data on inter-item correlations. Students receiving Form 2A (73 eleventh graders and 92 twelfth graders) responded to all problems in elementary algebra, intermediate/advanced algebra, and coordinate geometry and functions. Those receiving Form 2B (52 eleventh graders and 74 twelfth graders) responded to all problems in plane and solid geometry, trigonometry, and coordinate geometry and functions. The use of two modified forms rather than one was necessitated by the fact that the students could not respond to all problems within the class period, which typically lasted about 40 minutes. Forms 1, 2A, and 2B were randomly assigned to students within each class, in the ratio 1:2:2. Forms 2A and 2B were used for the majority of students in order to provide the largest possible sample for analysis of inter-item correlations.

In addition to completing the inventory, the students were asked to provide self-reported grades (on a five-point letter scale from A through F) in courses described as follows: "first course in algebra," "second course in algebra," "third course in algebra," "geometry (plane and, if included, solid)," and "trigonometry."

Results

Time to complete the inventory

One of the first questions of interest concerned the amount of time required by the students to complete the inventory. This question was posed only to those students who received Form 1. For the 52 students for whom data were

2. The version of the inventory shown in Appendix E incorporates the relatively minor modifications made as a result of the Phase II activities. The principal difference is that, in the version administered to the students, the students were asked "How successful were you at solving problems in this topic when you studied it?" (See Appendix D, "Assumptions and Explanation Regarding the Mathematics Inventory and Prototype Reports.") Other differences are that the version administered to the students included only 59 topics instead of the 64 listed here (some topics were later subdivided); it contained a few problems that were later omitted on the basis of redundancy with other problems; and it did not contain a few problems that were later added by suggestion of the consultants.

available (some students failed to fill in the times they began and/or completed the inventory), the median time to complete the inventory was 17 minutes, with a range of 5 to 35 minutes. (When the two students at each of the low and high extremes were excluded, the range was 10 to 30 minutes.)

Comparison of mean scores for Form 1 versus Forms 2A and 2B

For each student given Forms 2A and 2B, the score per topic was derived by averaging the student's responses to all items (that is, sample problems) within that topic. A question of interest was whether these students' average responses per topic were comparable to the topic responses given by students administered Form 1. The data suggested that this was the case. There were 43 topics in the inventory for which there were two or more sample problems. For each such topic, the average response per topic was computed for subjects given Forms 2A and 2B. Comparison of these averages with the topic responses of subjects given Form 1 showed a difference of only .07 points in favor of those given Forms 2A and 2B. Since the average S.D. of topic repsonses to Form 1 was .97, this difference represented only seven percent of the average S.D. of topic responses. Thus, the difference in topic responses between these two formats appeared to be relatively small.

Relations with other measures

Although the validity of the inventory remains to be determined in later research, some preliminary information regarding relations with other measures was available from this project. Specifically, correlations were computed between inventory responses and (a) SAT scores; (b) actual grades; (c) self-reported grades; and (d) average of the six self-reported grades on the SDQ.

Although correlations were computed separately for eleventh and twelfth grade students, those involving the twelfth grade students were of greatest interest, as these students had had more complete exposure to the basic mathematics sequence (which typically extends through eleventh grade). Thus, only the correlations for the twelfth grade students will be reported here. Since the number of twelfth graders administered Form 1 was too small for adequate interpretation of the correlational data, only the data for students administered Forms 2A and 2B were examined. Five scores were derived from the inventory, one for each of the major subject areas. (Scores were derived by averaging items within each topic and then computing an unweighted average across topics.) Inventory scores were found to correlate higher with SAT-mathematical score (*r*s ranged from .45 to .56 across the five subject areas) than with SAT-verbal score (*r*s ranged from .21 to .37). Of the correlations between inventory scores and self-reported grades, those involving inventory scores in elementary algebra and intermediate/advanced algebra were the highest (ranging from .59 to .71). The inventory score in geometry correlated higher with self-reported geometry grade (.53) than with the self-reported grades in the other four subjects (*r*s ranged from .20 to .27). The inventory score in trigonometry correlated highest with the self-reported trigonometry grade (.57), although the correlation with third-year algebra grade was nearly as high (.55); the other correlations ranged from .30 to .43. The inventory score in coordinate geometry correlated highest with self-reported second-year algebra grade (.56), which might be expected since topics in coordinate geometry are similar to those in advanced algebra, although it correlated nearly as highly with other self-reported grades (*r*s = .41 to 49).

Correlations were also computed between inventory scores and the composite of the six self-reported grades on the SDQ. These correlations ranged from .33 (with intermediate/advanced algebra inventory score) to .53 (with trigonometry inventory score).

Correlations with the students' actual grades were lower than those with the other measures. Correlations with first-year algebra grade ranged from −.20 to .36; correlations with second-year algebra grade ranged from .00 to .31; and correlations with geometry grade ranged from .18 to .44. The correlations in this case may have been attenuated by the fact that grades were taken from seven different grading systems (although the same differences in grading systems should also apply to the self-reported grades). Data from a much larger and more diverse sample are needed in order to establish the relationship between responses to the present inventory and students' performance in mathematics courses.

In summary, scores on the Mathematics Inventory correlated more highly with the mathematical than the verbal sections of the SAT; they correlated moderately highly with self-reported grades, and in many cases the highest correlation between an inventory score and a self-reported grade involved the same subject area. Although the relationships of inventory responses to actual grades were not as high, further study of these relationships is needed.

Correlations among subject areas of the inventory

Correlations among inventory scores for the five subject areas were also examined for the twelfth graders; in this case, only data from Form 1 were used, since only this form provided the full matrix of intercorrelations. It should be noted that the *N*s for these correlations were relatively small (42 students for correlations involving the first three subject areas, and 38 for those involving the last two, as four students chose not to complete the last two sections). The correlations ranged from .58 to .81, with a median of .72. These correlations are reasonably high and demonstrate a relatively strong relationship in students' estimates of their knowledge of the different subject areas. At the same time, the correlations were well below maximum. Once the reliabilities of inventory responses per subject area have been established with a more substantial sample, it will be

possible to draw conclusions about the degree of independence among scores per subject area.

Correlations among inventory topics

Separate correlation matrices were derived for eleventh and twelfth graders; for each grade, one matrix was computed for students given Form 1, and one for students given Forms 2A and 2B combined. Results for twelfth graders given Form 1 will be highlighted; results for the other analyses were generally similar. In general, the correlations were not exceedingly high. Only 1 of 66 inter-topic correlations in elementary algebra reached .70; this was true of 2 of 171 correlations in intermediate/advanced algebra; 3 of 55 correlations in geometry; 8 of 21 correlations in trigonometry; and 4 of 45 correlations in coordinate geometry and functions. Thus, the greatest amount of interrelationship among topics was found in the trigonometry area (this same observation was true for eleventh graders given Form 1, and for both eleventh and twelfth graders given Forms 2A and 2B). Note that this result could be as much a function of differential reliability of different sections as of differences in content. Despite this observation, however, analysis of the content of the trigonometry items indicated that no topic should be omitted, as each topic seemed to relate to a somewhat different area of knowledge. Hence, no changes were made in the inventory as a result of the inter-topic correlations.

Correlations among items within topics of the inventory

The observed intercorrelations among items (that is, sample problems) were expected to aid in revision of the inventory—particularly to help identify areas in which redundancy could be eliminated. Only the data for Forms 2A and 2B were relevant for examining these correlations. Correlations were computed separately for eleventh and twelfth graders (students given Forms 2A and 2B were included in a single correlational analysis per grade, with provision for missing data). Only the data for the twelfth grade students are presented here, since this group had had the most complete exposure to the mathematics coursework. The *N*s per correlation ranged from 88 to 92 for elementary and intermediate/advanced alegebra; from 72 to 74 for geometry and trigonometry; and from 141 to 147 for coordinate geometry and functions. Whenever a topic had two or more sample problems, the intercorrelations among problems with that topic were examined. Of 173 possible correlations, only 34 exceeded .80. Thus, according to the data from this limited sample, there did not appear to be an excessive amount of redundancy among sample problems (although, again, the magnitudes of the correlation are partly dependent on the reliabilities of the item responses; these reliabilities would need to be established with a much larger sample). In some cases in which extremely high correlations were observed between two problems within a topic, and examination of the content indicated duplication in areas of knowledge being measured, one of the problems was eliminated. Only four of the 147 problems were eliminated on this basis.

ANALYSIS OF STUDENTS' RESPONSES TO QUESTIONNAIRE

A questionnaire was attached to the inventory for students to complete after they finished the inventory if time permitted. Almost all of the students given Form 1, but few of the students given Forms 2A and 2B, had time to respond to the questionnaire. For this reason—and because Form 1 was the standard form of the instrument—only the responses of the students receiving Form 1 were examined. Responses of both eleventh and twelfth grade students combined will be reported here, since it was felt that the students' questionnaire responses, unlike their inventory responses, would not be greatly influenced by the amount of mathematics experience they had had.

A total of 60 students completed the questionnaire (the 12 students given Form 1 in the first school were not given the questionnaire; each of the other six schools contributed between 8 and 13 students to this sample). The questionnaire sample included 9 eleventh grade boys, 17 eleventh grade girls, 13 twelfth grade boys and 21 twelfth grade girls. The mean SAT-mathematical score of this sample was 550 and the mean SAT-verbal score was 489. Average (actual) grades in first-year algebra, second-year algebra and geometry, respectively (on the scale described above) were 6.1, 5.8 and 5.4. Thus, the questionnaire sample was similar to the total sample in SAT scores and only slightly below the total sample in mathematics grades.

The questionnaire given the students is shown in Appendix C, along with the data obtained with the questionnaire. The most salient aspects of the questionnaire analysis were as follows.

First, in the questionnaire it was suggested to the student that any of three categories of information might be provided in a report to a student who completes the inventory: (a) how the student's (self-reported) preparation compares with expectations of high school mathematics teachers nationwide; (b) how the student's preparation compares with that of other students completing the questionnaire; and (c) how the student's preparation compares with what he or she needs to know in order to take certain courses at colleges the student specifies. When asked how useful each of these types of information would be, the modal response in the first two cases was "probably useful" (the second highest category on a five-point scale from "definitely not useful" to "definitely useful"), whereas the modal response in the third case was "definitely useful."

The students were also told in the questionnaire to imagine that the inventory were offered as a regular service, and they were asked if they would be likely to complete the

questionnaire at each of various possible times: (a) when registering to take the SAT; (b) within a few weeks after registering for the SAT; (c) in the spring of junior year; and (d) in the spring of senior year. The modal response in each case was "probably would," the second highest level on a five-point scale from "definitely would not" to "definitely would." When asked at what time a report based on the students' inventory responses would be most useful, the majority of the students indicated the junior year of high school rather than the senior year or just before beginning the freshman year of college. When asked whether it would be important that their inventory report be sent only to them and not to a college or school, the majority of students indicated that it "definitely would" be important.

In sum, the students seemed to feel that the inventory could be useful, particularly (a) if it could provide information about readiness for college courses, (b) if it could provide such information well before completion of high school, and (c) if the students were the sole recipients of individualized information based on their inventory responses.

STUDENT INTERVIEWS

A total of 33 students were interviewed by the first author—five from each of five schools, and four from each of the other two schools. In four schools, interviews were conducted either in one group with all four or five students attending; in the other three schools, students were interviewed in two sessions of one to four students each. Each interview occupied a full class period of approximately 40 minutes and was conducted after the interviewees had completed the inventory.

Teachers selected students, or students volunteered, to participate in the interviews; random selection was not possible, as the interview required absence from another class. The sample of interviewees included 15 students in grade 11 and 16 in grade 12; it also included two students from grade 10 taking an eleventh grade class. (Tenth graders were eliminated from the data analyses described earlier; however, at the time the interviews were conducted and the interview data were analyzed, it was not known that tenth grade students were present, so their data are included here.) Interviewees included 13 boys and 20 girls. Their average SAT-mathematical score was 571, and their average SAT-verbal score was 495. Average actual grades in first-year algebra, second-year algebra and geometry, on the scale described above, were 6.2, 6.9 and 6.3, respectively. Thus, the interview sample tended to be somewhat more proficient on average than the general sample of students.

In order to obtain the most information possible from the interviews, students in all schools were not asked a uniform set of questions. Rather, any given question was posed to the students in approximately half the schools.

Response Scale

A question of major interest concerned the scale on which the students made their responses (that is, "very successful," "usually successful," etc.). The students were asked, "Given a scale from 0 to 10, with 0 meaning least successful and 10 meaning most successful (e.g., 0 meaning you got 0 problems right out of 10, and 10 meaning you got 10 right out of 10), what range of numbers on the scale from 0 to 10 would you associate with the following phrases: 'very successful,' 'usually successful,' 'sometimes successful,' and 'not successful'." In order to obtain independent judgments, students were asked to write their answers secretly on slips of paper. For each school, the median of the top and bottom figure assigned to each of the four ranges was computed. The resulting figures associated with the above-mentioned four phrases proved to be the same for all three schools: 10-9, 8-6, 5-3, and 2-0, respectively. These data suggest that the phrases that were selected for use in the inventory represent relatively discriminable levels of proficiency and that there is consistency in interpretation of these phrases.

The students were also asked to assign ranges of numbers (on a scale from 0 to 10) associated with the words "excellent," "good," "fair," and "poor." The results were almost identical to those for the phrases "very successful," and so on. Thus, although a change in phrases defining the inventory scale was under consideration at the time of the interviews, these data suggest that the original phrases were satisfactory.

Students in these three schools and a fourth school were also asked for information that would help relate the scale categories to the school grading system. In two of the schools, the students were simply asked to assign ranges of numbers (on a scale from 0 to 10) to the grades A, B, C, D and F. Since students in the other two schools were graded on a numerical system from 0 to 100, these students were asked to relate the terms "very successful," and so on, to numbers on their grading scale. For the latter two schools and one of the former two schools, the data indicated that the four categories of the scale ("very successful," etc.) translated roughly into the grades A, B, C, and combined D and F. For the fourth school, a problem with this exercise prevented interpretation of the data. In general, the results indicate that the response categories used in the inventory have relatively concrete referents in the students' grading systems.

It should be noted that, in scoring the inventory, the category "no experience" (did not study topic) was treated as a level below "not successful." In discussions with consultants and with staff members at Educational Testing Service, it was decided that it was appropriate to assign scores in this fashion. It was reasoned that if a student studied a topic and judged himself or herself "not successful," the student probably still acquired more information about that topic than if not exposed to the topic

at all. (More on this issue in Appendix D, "Assumptions and Explanation Regarding the Mathematics Inventory and Prototype Reports.")

The Self-Evaluation Process

Another issue discussed with the students concerned their ability to evaluate themselves retrospectively—that is, to recall how well they had learned a topic at the time they studied it (rather than how well they know it right now). This issue was discussed with students in three schools. Some students indicated that they answered the questions in the inventory with respect to their current knowledge. One student said he tried to effect a compromise between his original knowledge and his current knowledge; others said they responded in terms of how well they had originally learned the material. The students suggested that the instructions to the inventory should stress the fact that the students are to indicate how well they knew the material when they studied it. They also suggested making clear how a student should respond if he or she knows the material better now (through review in another course) than when it was first studied. In developing the version of the inventory shown in Appendix E, the instructions were modified to accommodate these suggestions.

CONSULTATION

The external consultants in Phase II of the project included the six persons who had served as consultants in Phase I plus four additional persons (see list of consultants in Appendix A). Phone conversations with these persons were conducted to examine the structure of the Mathematics Inventory and to help conceptualize the nature of the report(s) to students and to schools that would result from the inventory. Several Educational Testing Service staff members with expertise in psychometrics and instrument development were also consulted during this phase to discuss the nature of the inventory and the reports.

The consultants were asked about the structure of the inventory and about ways in which they felt it could be improved. One issue of interest concerned the topic definitions. Discussion with the consultants resulted in some reorganization of the inventory's structure, with a few new topics also added.

Some of the discussion with the consultants concerned the nature of the report to the student. There was general agreement that the various topics in the inventory should be appropriately weighted, so that a student's overall score for a given subject area would adequately reflect the relative importance of the various topics contributing to that score. (For example, "operations with signed numbers," "solving linear equations," and the other topics should be weighted appropriately in deriving a student's score in elementary algebra.) Methods of accomplishing the weighting process were discussed with several of the consultants; the most promising method that was suggested was to ask a large sample of teachers to distribute 100 percentage points among the various topics in a subject area such that the weights so assigned would produce an appropriate overall score for that subject area, just as one would choose a certain number of items per topic in constructing a 100-item test. (In the prototype reports presented in the appendixes, reference is made to a special manual, to be made available to secondary school guidance counselors, that would show these importance ratings and explain the computation of the student's inventory score per subject area as a weighted average.)

Another aspect of the student report that was discussed with the consultants concerned a comparison of the student's self-reported knowledge with prerequisites for taking certain college mathematics courses. This aspect is not included in the prototype report presented below, but it is mentioned here because the consultants expressed some interest in inclusion of such a report. Problems mentioned by the consultants, however, include (a) the difficulty of making a general statement about prerequisites among all colleges, or even among groups of colleges; and (b) changes in prerequisites over time, requiring continual updating of the information base. Nevertheless, if the inventory were to be used for purposes of placement after a student had selected a college, a report of the student's standing in relation to prerequisites for taking certain mathematics or science courses at that specific college could be a useful aspect of the student report.

Some consultants raised the issue of changing emphases in mathematics education—for example, a general trend in the field toward greater emphasis on preparation in finite (discrete) mathematics and less emphasis on the traditional calculus-related preparation, which is due partly to the increase in use of computers. Thus, reexamination of the inventory would be needed within a few years (and, indeed, probably at regular intervals) to ensure that it reflects the current thinking in the field. For the present, however, the inventory was regarded by the consultants as providing reasonably good coverage of the first three years of today's typical secondary school mathematics curriculum.

Many other suggestions and comments were also made by the consultants in either Phase I or Phase II regarding appropriate use of the inventory and interpretation of the data obtained from it. It was noted, for example, that a self-evaluation instrument such as the Mathematics Inventory can be a very useful tool, bearing in mind that the best index of a student's current proficiency in mathematics would be a test requiring students actually to solve problems.

It was pointed out that, among its uses, the inventory could serve a particularly valuable function in the process of course placement. For example, the inventory data might be used by a guidance counselor or advisor, perhaps as a preliminary step before administration of a placement test. It was acknowledged, however, that the role of the inventory

in the placement process could differ considerably among institutions.

Also mentioned was the importance of presenting the inventory in a way that ensures that students represent their mathematics preparation as accurately as possible. A student could conceivably either overestimate or underestimate his or her preparation, depending on the intended use of the inventory or on characteristics of the individual student.

SUMMARY AND INTERPRETATION

Certain conclusions have emerged from the pilot work with the draft inventory. First, the limited data obtained with this small sample indicated very few instances of strong inter-item redundancy that would demand a further condensation of the inventory. Where the inventory has been revised since completion of the pilot work, this revision generally has involved subdivision of topics, along with addition of a few new topics and deletion of others, based principally on discussions with the consultants.

The time required to complete the inventory (which presumably would not change substantially as a result of the revisions made) appears to range from about 10 to 30 minutes for almost all students, with an average of about 17 minutes. Preliminary evidence suggests that the inventory responses may be at least moderately related to certain other indexes of mathematics proficiency; however, the data are extremely tentative, given the small sample used here, and conclusions about the concurrent validity of the inventory cannot be drawn without a more comprehensive validation effort.

The interview responses suggested that the scale used to obtain the students' responses is a reasonable one but that certain changes in the instructions to the inventory could improve the instrument. (The instructions shown in the prototype inventory in Appendix E accommodate the students' most critical comments in this regard.) The questionnaire results indicated that students see the inventory as useful and tend to regard the junior year of high school as the time the information from the inventory would be most helpful.

Regarding the format of the report that is sent to the student, discussions with consultants indicated that it is desirable to obtain importance ratings per topic so that proper weight can be given to each topic in deriving the student's overall score in a given subject area.

FINAL REMARKS

Included in the appendixes to this report are: (a) the Mathematics Inventory, with instructions and answer sheet; (b) a prototype report to the student based on his or her responses to the inventory; and (c) a prototype report to the secondary school. These report formats have been developed based on some assumptions about the most likely uses of the inventory. Other report formats are also possible, however, and development of the best methods of reporting the inventory data to the students and to the schools is expected to be a continuing process, based partly on input from potential institutional users regarding the kinds of information they and their students would find most valuable.

A final word is in order about subsequent activities in development of the Mathematics Inventory and procedures for its implementation. In order for the inventory to be used appropriately, it is necessary first to obtain the importance ratings described above—that is, the weights assigned to the various topics that would be used in computing the student's score for each subject area. Also, if the inventory is to provide information of a normative type, it is necessary to collect data from an appropriate norm group, so that comparisons with this norm group can be made (see Appendix D, "Assumptions and Explanation Regarding the Mathematics Inventory and Prototype Reports").

Additionally, validation efforts are essential. As indicated in the first footnote to this report, the Mathematics Inventory presented here is still regarded as an experimental instrument and has not yet been subjected to proper validation. Validation, in essence, requires assessment of whether an instrument serves its intended purpose. It is hoped that, as various institutions or groups use the inventory, they will collect data that can help determine whether the inventory adequately serves the purposes for which it is used. Compilation of statistics from a number of users can provide a cumulative data base, from which general conclusions about the validity of the instrument can be drawn.

APPENDIX A. EXTERNAL CONSULTANTS

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APPENDIX B. SECONDARY SCHOOLS PARTICIPATING IN PILOT STUDY AND PROJECT COORDINATORS

School	*Coordinator(s)*
Cardinal Spellman High School New York, New York	Sister Winifred Goddard
Freehold Township High School Freehold, New Jersey	Mrs. Ethel Rubinstein Mr. Thomas McNamara
Hopewell Valley Central High School Pennington, New Jersey	Ms. Elene Van Noy
Madison Central High School Old Bridge, New Jersey	Mrs. Ethel Newman
Ocean City High School Ocean City, New Jersey	Mr. George Brock
St. Joseph's Preparatory School Philadelphia, Pennsylvania	Mr. Daniel Brennan Mr. Albert Zimmerman
West Orange High School West Orange, New Jersey	Miss Lillian Kunz

APPENDIX C. STUDENT QUESTIONNAIRE AND DATA FROM STUDENTS COMPLETING THE QUESTIONNAIRE

Following is the questionnaire as it was administered to the students. The number of students choosing each response option is presented below the response option. (Note that all students did not respond to all questions.) Questions 1, 2, and 7 had included an "other" category, but the responses to this category are not presented, as only a very rare student responded to this category, and the information provided was marginally useful. Also, an open-ended question at the end of the questionnaire asking for general reactions and suggestions has been omitted here. The responses to the open-ended question were relatively short, given the time constraint, and they provided little information beyond that already provided by the responses to the other questions. Questions about the students' educational plans have also been omitted here.

1. The report a student would receive based on this questionnaire could contain several different types of information. How useful to the student do you think each of the following types of information would be? (Circle one of the underlined phrases in each case.)

a. How the student's (self-reported) preparation compares with expectations of high school mathematics teachers nationwide.

definitely useful	probably useful	neutral	probably not useful	definitely not useful
17	24	13	5	0

b. How the student's preparation compares with that of other students completing the questionnaire.

definitely useful	probably useful	neutral	probably not useful	definitely not useful
18	26	10	4	0

c. How the student's preparation compares with what he or she needs to know in order to take certain courses at colleges the student specifies.

definitely use[illegible]	probably useful	neutral	probably not useful	definitely not useful
44	12	2	0	0

For Questions 2 through 8, imagine that the questionnaire is being offered as a regular service and that you have an opportunity to complete the questionnaire and to receive a report based on your responses to the questionnaire.

2. Pretend that you are just now applying to take the SAT, and you are deciding whether to complete the questionnaire. Which of the following types of information *must the report contain* in order for you to decide to complete it? (Check all that apply.)

   25 How your (self-reported) preparation compares with expectations of high school teachers nationwide

   30 How your preparation compares with that of other students completing the questionnaire

   53 How your preparation compares with what you need to know in order to take certain courses at colleges you specify

3. Assume the report will contain the type of information you indicated in Question 2. Would you complete the questionnaire when registering to take the SAT? (circle one)

definitely would	probably would	neutral	probably would not	definitely would not
22	29	2	2	0

4. If the questionnaire could be completed any time within a few weeks after registering for the SAT, would you complete it? (Again, assume that the report will contain the type of information you indicated in Question 2.) (circle one)

definitely would	probably would	neutral	probably would not	definitely would not
20	31	3	1	0

5. If the questionnaire were to be completed in the spring of junior year, would you complete it (if you are not a junior, answer this question as if you were now a junior). (circle one)

definitely would	probably would	neutral	probably would not	definitely would not
15	29	8	3	0

6. If the questionnaire were to be completed in the spring of senior year, would you complete it? (circle one)

definitely would	probably would	neutral	probably would not	definitely would not
7	25	6	14	3

7. At which of the following times would you find it *most* useful to have the type of information that would be provided in the report based on the questionnaire? (check one)

   a. 20 at the beginning of junior year

   b. 20 toward the end of junior year

   c. 12 at the beginning of senior year

   d. 2 toward the end of senior year

   e. 1 just before the beginning of the freshman year of college

8. In deciding whether to complete the questionnaire, would it be important that your individual report be sent *only to you* and not to any college or school? (circle one)

definitely would	probably would	neutral	probably would not	definitely would not
29	16	8	1	1

APPENDIX D. ASSUMPTIONS AND EXPLANATION REGARDING THE MATHEMATICS INVENTORY AND PROTOTYPE REPORTS

Before presentation of the Mathematics Inventory and prototype reports in Appendixes E, F, and G, it is essential to provide some explanation and to make explicit certain underlying assumptions.

Mathematics Inventory

The Mathematics Inventory (shown in Appendix E) asks the students how well they knew the material in each subject area at the time they knew it best. This is more likely to have been true when the students took the relevant course, in the case of some subjects such as geometry, since material

learned in geometry is not likely to have been reviewed in subsequent courses. However, for material learned in other subjects such as elementary algebra, this is more likely to be true at the time the students complete the inventory (if they complete it during the junior or senior year of secondary school); many students will have just taken, or will currently be taking, other courses such as advanced algebra which build on the concepts learned earlier and strengthen the student's knowledge of them.

It was felt that the inventory would be most useful if it ascertained students' knowledge of a topic when they knew it best, rather than how well they know it now. If judgments are to be made about a student's readiness for a course requiring knowledge of a certain subject area, it is most useful to know that the student had a good grasp of the subject area at one time so that, presumably, some review would readily bring the student back to his or her former level of knowledge. This student, whose knowledge of a topic may be poor at present, must certainly be differentiated from the student whose current knowledge is poor due to never having learned the topic.

Another important consideration is that students should examine all sample problems in a topic and, if the problems differ in complexity, the students should give responses that show, on balance, their command of the topic. Some thought was given to providing more detailed instructions in this regard. However, two factors mitigate against doing so. First, the data reviewed above show that students generally base their responses on all sample problems: for a given topic the average response to the sample problems for that topic by students given Forms 2A and 2B was about the same as the single response to that topic by students given Form 1. Second, it seemed desirable not to induce students to be overly meticulous by implying that they should spend time computing a precise average of their judgments about the individual problems.

Report to the Individual Student

The proposed report to the student (shown in Appendix F) has two aspects—a criterion-referenced aspect and a norm-referenced aspect. The criterion-referenced aspect is the set of Xs in the table showing the student's average score on the inventory and the portion of the narrative listing the topics in which the student rated himself or herself as neither "very successful" nor "usually successful." The norm-referenced aspect consists of the vertical lines in the table presenting the 50 percent range for all students completing the inventory and the statements in the narrative indicating topics in which the student falls below or above the 50 percent range for all students completing the inventory.

The criterion-referenced aspect, in itself, can serve as a tool to aid the student in educational planning and to assist in discussing the student's areas of strength and weakness with an advisor. The norm-referenced aspect presents additional information that can also be helpful, assuming that the norm group provides a useful standard against which the student may compare himself or herself. Ideally, the students in the norm group would comprise a reasonable cross-section of college-bound students.

The listing of topics in which the student rated himself or herself below "usually successful" does not involve any data manipulation, but it is regarded as useful for the student. It reminds the student of his or her responses and thus provides, in capsule form, a list of topics in which the student may be deficient in preparation. This list can be used to facilitate discussion with a counselor or a college advisor.

Much discussion has been conducted about the role of the category, "did not study topic," in the response scale. It could be argued that the student who did not study a topic is a more capable student than one who attempted to learn a topic and failed; presumably the former student, if given the opportunity, might learn the material well. However, the objective here is to ascertain the degree to which a student has had a command of each of several mathematics topics. In this context, the student who has had no experience with a topic knows nothing about it, and his or her response must be scaled accordingly. Thus, in the prototype report presented here, "did not study topic" is treated as a category below "not successful." The decision to order these two categories in this way is based partly on the assumption that, even if a student indicates a lack of success in learning a topic, the fact that the student has studied the topic at all means that he or she may well know more about it than the student who has had no exposure to the topic at all.

It will be noted that the information presented in the first table of the student report and all of the narrative information following it except the lead-in statement is individualized. It is envisioned that this information would be generated and printed by computer for each individual student.

The examples in the prototype report here show feedback by individual topics. However, another possibility is to provide feedback by clusters of topics. It remains an issue for discussion which of these approaches would be most useful and appropriate. Ultimate resolution of this issue will depend partly on input provided by potential users of the inventory.

Students should be instructed, when they complete the inventory, to keep a copy of the inventory. This copy can then serve as a reference when the students receive their reports.

Report to the School

As is true of the report to the student, the prototype report to the school (shown in Appendix G) has a criterion-related aspect and a norm-referenced aspect. The criterion-related aspect consists of information about the degree to which the students in a given school (according to their own self-

report) have learned the essential material in the basic mathematics sequence, and it provides a breakdown showing topics in which the school's students might need improvement. The norm-referenced aspect shows how the school's students compare with those in other schools. The value of the latter information will depend on the establishment of an appropriate norm group.

The types of information that could be presented to the secondary school are even more varied than those that could be presented to the individual student. The kinds of information shown in the prototype report here are those that are believed to be particularly useful. However, continued discussion with school personnel who would use this information can help specify schools' informational needs and, thus, aid in determining the most useful format for a report to the school.

A copy of the Mathematics Inventory could be included as part of the report to the school, which would serve as a reference. In addition, it is proposed that a manual also be included—referred to above as the "Mathematics Profile Manual"—which would help explain how students' individual scores are computed and thus provide the school's guidance counselor with a resource for helping answer students' questions.

APPENDIX E. MATHEMATICS INVENTORY

Directions for Completing the Mathematics Inventory

This inventory deals with five subject areas: (a) elementary algebra; (b) intermediate/advanced algebra; (c) plane and solid geometry; (d) trigonometry; and (e) coordinate geometry and functions. Listed under each subject area are several topics; these are numbered from 1 to 64. Under each topic name are sample problems. You are not required to work the problems; they are presented here merely to illustrate the topics. Be sure to base your answers on examination of *all* the problems illustrating a topic.

For each topic, answer the following question:

How successful were you at solving problems in this topic when you knew it best?

Very Successful—I could *always (or almost always)* solve this type of problem

Usually Successful—I could *usually* solve this type of problem

Sometimes Successful—I could *sometimes* solve this type of problem

Not Successful—I could *never (or rarely)* solve this type of problem

I did not study this topic

Note. You probably knew the material best at the time you studied it, as you may have forgotten some of it since then. But if you know a topic best *now,* due to review in a later course, base your answer on your present knowledge.

Mark your responses on the answer sheet by blackening the appropriate ovals.

The answer sheet for the inventory begins on the next page.

APPENDIX E continued

Answer Sheet for Mathematics Inventory

For each topic: How successful were you at solving problems in this topic when you knew it best? Blacken the appropriate oval in each case

	Very Successful	Usually Successful	Sometimes Successful	Not Successful	Did Not Study Topic
Elementary Algebra					
1 operations with signed numbers	0	0	0	0	0
2 substitution	0	0	0	0	0
3 simplifying expressions with nonnegative integer exponents	0	0	0	0	0
4 simplifying polynomial expressions	0	0	0	0	0
5 factoring polynomial expressions—elementary	0	0	0	0	0
6 solving simple equations in one unknown	0	0	0	0	0
7 solving inequalities: linear/other elementary	0	0	0	0	0
8 graphing linear equations	0	0	0	0	0
9 solving systems of two linear equations	0	0	0	0	0
10 translations and word problems	0	0	0	0	0
11 simplifying radicals	0	0	0	0	0
12 simplifying rational (fractional) expressions—concepts	0	0	0	0	0
Intermediate/Advanced Algebra					
13 evaluating expressions with negative integer exponents	0	0	0	0	0
14 evaluating expressions with fractional exponents	0	0	0	0	0
15 operations involving radicals	0	0	0	0	0
16 factoring polynomial expressions—advanced	0	0	0	0	0
17 finding polynomial equations	0	0	0	0	0
18 operations with rational expressions—advanced	0	0	0	0	0
19 operations with complex numbers	0	0	0	0	0
20 logarithms—concepts	0	0	0	0	0
21 solving fractional equations	0	0	0	0	0
22 solving simple radical equations	0	0	0	0	0
23 solving exponential equations	0	0	0	0	0
24 solving logarithmic equations	0	0	0	0	0
25 solving literal equations	0	0	0	0	0
26 solving quadratic equations	0	0	0	0	0
27 solving inequalities: quadratic/other advanced	0	0	0	0	0
28 graphing linear equations; finding slope and intercepts	0	0	0	0	0
29 graphing systems of linear inequalities	0	0	0	0	0
30 graphing quadratic equations and inequalities	0	0	0	0	0
31 solving systems of equations—advanced	0	0	0	0	0
32 binomial expansion	0	0	0	0	0
33 sequences and series	0	0	0	0	0

	Very Successful	Usually Successful	Sometimes Successful	Not Successful	Did Not Study Topic
Plane and Solid Geometry					
34 parallel lines	0	0	0	0	0
35 triangles—general	0	0	0	0	0
36 right triangles	0	0	0	0	0
37 applications of the Pythagorean Theorem	0	0	0	0	0
38 interior angles of polygons	0	0	0	0	0
39 circles and inscribed figures	0	0	0	0	0
40 deductive reasoning and proof	0	0	0	0	0
41 rectangular solids	0	0	0	0	0
42 right circular cylinders and cones	0	0	0	0	0
43 spheres	0	0	0	0	0
Trigonometry					
44 angle measure	0	0	0	0	0
45 triangles/angles	0	0	0	0	0
46 equations	0	0	0	0	0
47 knowledge of identities	0	0	0	0	0
48 proof of identities	0	0	0	0	0
49 inverses	0	0	0	0	0
50 graphs	0	0	0	0	0
51 domain and range	0	0	0	0	0
52 applications of the Law of Sines and the Law of Cosines	0	0	0	0	0
Coordinate (Analytic) Geometry and Functions					
53 midpoint formula	0	0	0	0	0
54 distance formula	0	0	0	0	0
55 lines	0	0	0	0	0
56 conics (circles, parabolas, ellipses, hyperbolas)	0	0	0	0	0
57 functions—evaluation	0	0	0	0	0
58 functions—arithmetic and composition	0	0	0	0	0
59 functions—inverses	0	0	0	0	0
60 functions—zeros	0	0	0	0	0
61 graphing absolute value functions	0	0	0	0	0
62 graphing polynomial functions	0	0	0	0	0
63 graphing rational functions	0	0	0	0	0
64 graphing exponential and logarithmic functions	0	0	0	0	0

Mathematics Inventory

ELEMENTARY ALGEBRA

1. Operations with Signed Numbers

Q. $-3 - (-9) =$
A. $+6$

Q. $9 \times (-3) =$
A. -27

2. Substitution (of values in algebraic expressions)

Q. Find the value of $a^2 - b^3$ when $a = 2$ and $b = -3$.
A. 31

3. Simplifying Expressions with Nonnegative integer Exponents

Simplify the following expressions.

Q. $5x^0 =$
A. 5

Q. $(2a^2b)(ab^2) =$
A. $2a^3b^3$

Q. $\dfrac{5x^2y^4}{15xy^6} =$

A. $\dfrac{x}{3y^2}$

Q. $\left(\dfrac{2x^2y}{3y^3x}\right)^4 =$

A. $\dfrac{16x^4}{81y^8}$

4. Simplifying Polynomial Expressions

Simplify the following expressions.

Q. $2x + y - 3(x - y) =$
A. $-x + 4y$

Q. $3x^3 - 5x^2 - 2(x^2 - 3x) =$
A. $3x^3 - 7x^2 + 6x$

Q. $(2x + 7)(2x - 7) =$
A. $4x^2 - 49$

Q. $(-3m + 8)(4m - 10) =$
A. $-12m^2 + 62m - 80$

5. Factoring Polynomial Expressions—Elementary

Factor each of the following completely.

Q. $9x^2y - 12xy^2 =$
A. $3xy(3x - 4y)$

Q. $a^2 - 36b^2 =$
A. $(a + 6b)(a - 6b)$

Q. $x^2 + 4x + 4 =$
A. $(x + 2)^2$

Q. $x^2 + 5x + 6 =$
A. $(x + 2)(x + 3)$

Q. $2x^3 + 10x^2 + 12x =$
A. $2x(x + 2)(x + 3)$

6. Solving Simple Equations in One Unknown

Solve for x:

Q. $3x + 3 = -2x + 7$
A. $x = \dfrac{4}{5}$

Q. $|x| = 4$
A. $x = 4$, or -4

Q. $x^2 - 5x + 4 = 0$
A. $x = 1$, or 4

Q. $\dfrac{5}{x - 3} = \dfrac{10}{x}$
A. $x = 6$

7. Solving Inequalities: Linear/Other Elementary

Solve for x:

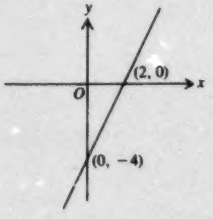
Q. $-4x < 5x - 5$
A. $x > \dfrac{5}{9}$

Q. $|x| < 3$
A. $-3 < x < 3$

8. Graphing Linear Equations

Q. Is $(2, -3)$ on the graph of the equation $x - y = 5$?
A. Yes

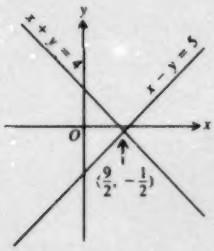
Q. Sketch the graph of the equation $2x - y = 4$.
A.



9. Solving Systems of Two Linear Equations

Q. Solve and graph the system: $\begin{cases} x + y = 4 \\ x - y = 5 \end{cases}$

A. $\left(\frac{9}{2}, -\frac{1}{2}\right)$



10. Translations from English to Algebraic Notation and Word Problems (Including Variation)

Q. Write an expression for the total dollar value of n nickels and d dimes.

A. $.05n + .10d$

Q. Mary purchased a car for \$7,500 and later sold it for \$5,000. Her loss was what percent of the purchase price?

A. $33\frac{1}{3}\%$

Q. There are 10 girls and 15 boys in a certain mathematics class. What fraction of the class is boys?

A. $\frac{3}{5}$

Q. What is the area of a rectangle with perimeter 20 if its length is 4 times its width?

A. 16 square units

Q. If x varies directly with y and $x = 5$ when $y = 7$, what is the value of y when $x = 25$?

A. 35

11. Simplifying Radicals

Q. Simplify $\sqrt{98}$.

A. $7\sqrt{2}$

12. Simplifying Rational (Fractional) Expressions—Concepts

Q. If $a \neq b$, then $\frac{a-b}{b-a} =$

A. -1

Q. For what value of x is the expression $\frac{3}{x-7}$ undefined?

A. 7

INTERMEDIATE AND ADVANCED ALGEBRA

13. Evaluating Expressions with Negative Integer Exponents

Q. $4^{-2} =$

A. $\frac{1}{16}$

14. Evaluating Expressions with Fractional Exponents

Q. Express $\sqrt{a^3}$ with a fractional exponent.

A. $a^{\frac{3}{2}}$

Q. $8^{-\frac{2}{3}} =$

A. $\frac{1}{4}$

15. Operations Involving Radicals

Perform the indicated operations.

Q. $\sqrt{50} \cdot \sqrt{30} =$

A. $10\sqrt{15}$

Q. $\sqrt{32} + \sqrt{50} =$

A. $9\sqrt{2}$

16. Factoring Polynomial Expressions—Advanced

Factor each of the following completely over the set of real numbers.

Q. $a^3 - b^3 =$

A. $(a - b)(a^2 + ab + b^2)$

Q. $3x^2 - 27x^2y^4 =$

A. $3x^2(1 + 3y^2)(1 + \sqrt{3}y)(1 - \sqrt{3}y)$

Q. $x^2 - 6xy - 4 + 9y^2 =$

A. $(x - 3y + 2)(x - 3y - 2)$

17. Finding Polynomial Equations

Q. Find a polynomial of least degree with zeros -2, 3, and -4.

A. $(x + 2)(x - 3)(x + 4) = x^3 + 3x^2 - 10x - 24$

18. Operations with Rational Expressions (i.e., Algebraic Fractions)—Advanced

Perform the indicated operations and write as a single fraction reduced to lowest terms.

Q. $\frac{3x}{(x+2)^2} - \frac{2x+1}{x^2-4} =$

A. $\frac{x^2 - 11x - 2}{(x+2)^2(x-2)}$

Q. $\left(\frac{a-5}{2(b+2)}\right) \cdot \left(\frac{4(b+2)^2}{a^2 - 25}\right) =$

A. $\frac{2(b+2)}{a+5}$

Q. $\left(\frac{2m+12}{n-2}\right) \div \left(\frac{m^2+7m+6}{2-n}\right) =$

A. $\frac{-2}{m+1}$

Q. $\frac{\frac{1}{x+1}}{1 - \frac{1}{x}} =$

A. $\frac{x}{x^2-1}$

19. Operations with Complex Numbers

Q. $(4 + i)(2 - 3i) =$
A. $11 - 10i$

Q. Write $\frac{2 + i}{3 - i}$ in the form $a + bi$.
A. $\frac{1}{2} + \frac{1}{2}i$

Q. $\sqrt{-2} \cdot \sqrt{-6} =$
A. $-2\sqrt{3}$

20. Logarithms—Concepts

Q. $\log_2 8 =$
A. 3

Q. $\log_{10} 2 + \log_{10} 5 =$
A. 1

21. Solving Fractional Equations

Q. Solve over the set of real numbers:
$1 - \frac{6}{x^2 - 1} = \frac{3}{x - 1}$
A. $x = -2$, or 5

22. Solving Simple Radical Equations

Q. Solve over the set of real numbers: $2 - x = \sqrt{x}$

Solution: $4 - 4x + x^2 = x$
$x^2 - 5x + 4 = 0$

A. 1 (4 is an extraneous root)

23. Solving Exponential Equations

Solve for x:

Q. $3^x + 1 = 28$
A. $x = 3$

Q. $2^{(x+1)} = 32$
A. $x = 4$

24. Solving Logarithmic Equations

Q. Solve for x: $\log_3 x = 3$
A. $x = 27$

25. Solving Literal Equations

Q. Solve for x, if $a = c$:
$ax + b = cx - d$
A. $x = \frac{b + d}{c - a}$

26. Solving Quadratic Equations

Q. Solve for x: $x^2 + 3x + 1 = 0$
A. $x^2 + 3x + \frac{9}{4} = -1 + \frac{9}{4}$

$(x + \frac{3}{2})^2 = \frac{5}{4}$

$x = -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$

or

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$

$x = \frac{-3 \pm \sqrt{5}}{2}$

Q. Solve for x over the set of complex numbers:
$x^2 + x + 1 = 0$
A. $x = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

Q. Use the discriminant to determine the nature of the roots of $x^2 - x + 1 = 0$.
A. $b^2 - 4ac = -3$
The roots are unequal and complex.

Q. Given that a, b, and c are real numbers, and that one solution of $ax^2 + bx + c = 0$ is $p + qi$, what is the other solution?
A. $p - qi$

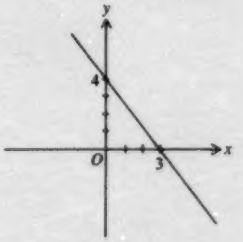
27. Solving Inequalities: Quadratic/Other Advanced

Q. Solve for x: $x^2 - 5x + 6 \geq 0$
A. $x \leq 2$ or $x \geq 3$

Q. Solve for x: $|x + 1| < |x + 2|$
A. $x > -\frac{3}{2}$

28. Graphing Linear Equations and Finding Slope and Intercepts

Q. Sketch the graph of the equation $4x + 3y = 12$, give the slope and the x- and y-intercepts of the graph.
A.



$m = -\frac{4}{3}$

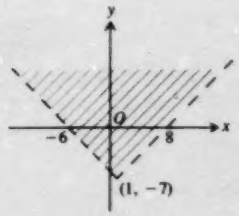
intercepts: (3, 0), (0, 4)

29. Graphing Systems of Linear Inequalities

Q. Graph the solution set of the system:

$$\begin{cases} x + y > -6 \\ x - y < 8 \end{cases}$$

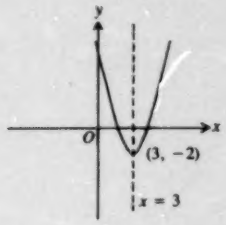
A.



30. Graphing Quadratic Equations and Inequalities

Q. Sketch the graph of the equation $y = (x - 3)^2 - 2$. Give the coordinates of the vertex, the x- and y-intercepts and an equation of the axis of symmetry.

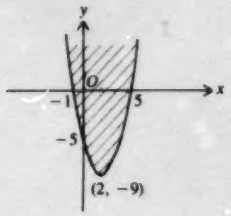
A.



vertex: $(3, -2)$
intercepts: $(0, 7)$, $(3 \pm \sqrt{2}, 0)$
axis $x = 3$

Q. Sketch the graph of $y \geq x^2 - 4x - 5$.

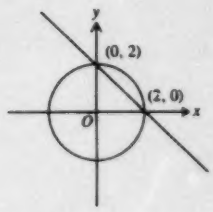
A.



31. Solving Systems of Equations—Advanced

Q. Solve and graph the system: $\begin{cases} x + y = 2 \\ x^2 + y^2 = 4 \end{cases}$

A. $(0, 2)$, $(2, 0)$



Q. Solve the system: $\begin{cases} 2x - y - z = 1 \\ 2x - 3y - 4z = 0 \\ x + y - z = 4 \end{cases}$.

A. $(1, 2, -1)$

32. Binomial Expansion

Q. Expand $(x - y)^4$.

A. $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

Q. What is the fourth term in the binomial expansion of $(x - 2y)^6$?

A. $-160x^3y^3$

33. Sequence and Series

Q. Given the arithmetic sequence $-10, -8, -6, \ldots$, and the expression for the nth term of an arithmetic sequence, $a + (n - 1)d$, what are the values of a and d?

A. $a = -10$
$d = 2$

Q. What is the seventh term of the arithmetic sequence whose first three terms are $5, 6\frac{1}{2}, 8, \ldots$?

A. 14

Q. What is the fifth term of the geometric sequence whose first three terms are $\frac{1}{4}, \frac{1}{2}, 1, \ldots$?
What is the common ratio in this sequence?

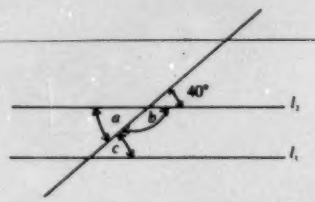
A. fifth term = 4
common ratio = 2

Q. What is the sum of the infinite geometric series whose first four terms are $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \ldots$?

A. $\frac{1}{3}$

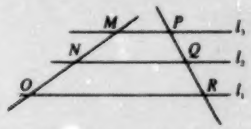
PLANE AND SOLID GEOMETRY

34. Parallel Lines



Q. If $l_1 \parallel l_2$ in the figure above, what are the measures of angles a, b, and c?

A. $a = 40°$
$b = 140°$
$c = 40°$



Q. If $l_1 \parallel l_2 \parallel l_3$ and if the length of $NM = NO = 5$ and the length of $PR = 7$ in the figure above, what is the length of QR?

A. $3\frac{1}{2}$

35. Triangles—General

Q. What is the area of a triangle with base 5 and height $5\sqrt{2}$?

A. $\frac{25}{2}\sqrt{2}$ square units

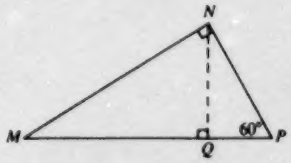
36. Right Triangles

Q. M and N are the acute angles of a right triangle. If the ratio of M to N is 4:1, what is the measure of angle M, in degrees?

A. 72°

Q. In ΔABC above, the length of $AB = 4$, $AC = 9$, and $DE = 3$. Find the length of CE.

A. $6\frac{3}{4}$



Q. In the triangle above, the length of $MP = 10$.
What is the length of MN?

A. $5\sqrt{3}$

Q. What is the length of of NQ?

A. $\frac{5}{2}\sqrt{3}$

Q. Which triangles are similar to ΔMNP?

A. ΔMQN, ΔNQP

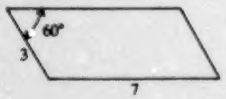
37. Applications of the Pythagorean Theorem

Q. If a rectangle has length 4 and width 3, what is the length of its diagonal?

A. 5

Q. What is the length of the diagonal of a square with perimeter 24?

A. $6\sqrt{2}$



Q. What is the area of the parallelogram above?

A. $\frac{21}{2}\sqrt{3}$ square units

38. Interior Angles of Polygons

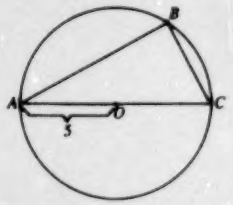
Q. What is the sum of the interior angles of a hexagon?

A. 720°

Q. What is the measure of an interior angle of a regular octagon?

A. 135°

39. Circles and Inscribed Figures

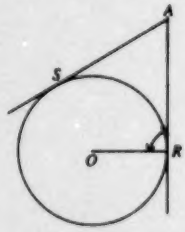


Q. O is the center of the circle above.
What is the area of the circle?

A. 25π square units

Q. What is the measure of $\angle ABC$?

A. 90°



Q. In the circle above, AS and AR are tangents and OR is a radius.
What is the measure of $\angle ORA$?

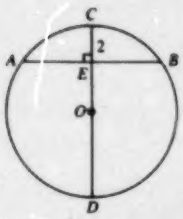
A. 90°

Q. If the length of $AS = 10$, what is the length of AR?

A. 10

Q. If minor arc SR is 120°, what is the measure of $\angle SAR$?

A. 60°



Q. In the circle with center O above, AB and CD are chords. If the length of $CD = 10$, what is the length of AE?

A. 4

40. Deductive Reasoning and Proof

Q. In a statement of the form "If a, then b," if b is true, then
(A) a is true
(B) a is false
(C) the truth value of a cannot be determined

A. (C)

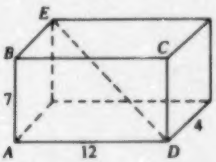
Q. If the statement "If not p, then not q" is known to be false, what can be said about the truth value of p?
(A) p is true.
(B) p is false.
(C) The truth value of p cannot be determined.

A. (B)

Q. In triangles ABC and ADC above, $AB = CD$ and $\angle BAC = \angle DCA$. Which of the standard congruence theorems or axioms would you use to prove that ΔABC is congruent to ΔCDA?

A. SAS

41. Rectangular Solids



Q. The figure above is a rectangular solid.
What is the volume of the solid?

A. 336 cubic units

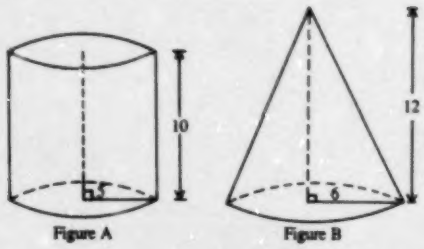
Q. What is the surface area of the solid?

A. 320 square units

Q. What is the length of diagonal DE?

A. $\sqrt{209}$ units

42. Right Circular Cylinders and Cones



Q. What are the volumes of the solids in the figures above?

A. Fig. A: 250π cubic units
Fig. B: 144π cubic units

Q. What are the surface areas of the solids in the figures above? (Include all bases.)

A. Fig. A: 150π square units
Fig. B: $36\pi + 36\pi\sqrt{5}$ square units

43. Spheres

Q. What is the volume of a sphere with radius 5?

A. $\frac{500}{3}\pi$ cubic units

Q. What is the surface area of this sphere?

A. 100π square units

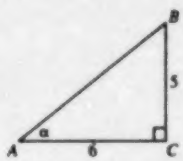
TRIGONOMETRY

44. Angle Measure

Q. $\frac{\pi}{6}$ radians is equivalent to what degree measure?

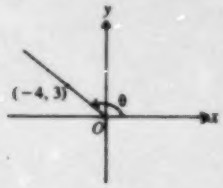
A. 30°

45. Triangles/Angles



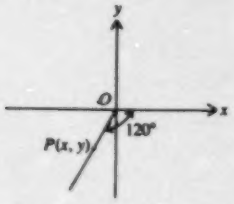
Q. In right triangle ABC above, what is the value of $\sin \alpha$?

A. $\sin \alpha = \frac{5}{\sqrt{61}}$



Q. In the figure above, what is the value of cos θ?

A. $\cos\theta = -\frac{4}{5}$



Q. In the figure above, if the distance from the origin to point P is 6, what are the coordinates of point P?

A. $(-3, -3\sqrt{3})$

46. Equations

Q. If $0 \leq \theta \leq 2\pi$, what are the values of θ such that $\sin\theta\cos\theta - \cos\theta = 0$?

A. $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

47. Knowledge of Identities

Q. Match each expression in the left-hand column with an equivalent expression in the right-hand column.

1. $\frac{\sin x}{\cos x}$	a. $\tan^2 x$
2. $\sec^2 x - 1$	b. $\sec x$
3. $\sin 2x$	c. $2\sin x\cos x$
4. $\frac{1}{\cos x}$	d. $\tan x$

A. 1—d
2—a
3—c
4—b

48. Proof of Identities

Q. Prove that the equation $(\sin x + \cos x)^2 = 1 + \sin 2x$ is an identity.

A. $(\sin x + \cos x)^2 =$
$\sin^2 x + 2\sin x\cos x + \cos^2 x =$
$(\sin^2 x + \cos^2 x) + 2\sin x\cos x =$
$1 + \sin 2x$

49. Inverses

Q. Evaluate Arccos $(\cos\frac{11\pi}{6})$.

A. $\frac{\pi}{6}$

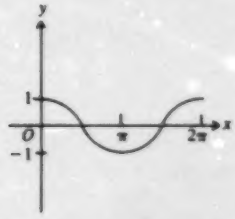
Q. If $\sin\theta = \frac{1}{2}$, what is the least possible positive value of θ?

A. $\frac{\pi}{6}$.

50. Graphs

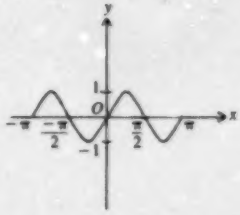
Q. Sketch the graph of $y = \cos x$ for $0 \leq x \leq 2\pi$.

A.



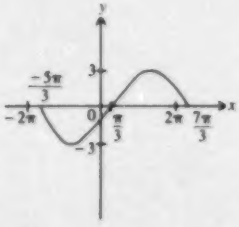
Q. Sketch the graph of $y = \sin 2x$ over 2 periods.

A.



Q. Sketch one cycle of the graph of the equation $y = 3\sin\frac{1}{2}(x - \frac{\pi}{3})$.

A.



Q. What is the period of the graph?

A. Period: 4π

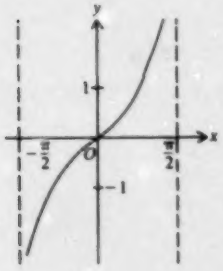
51. Domain and Range

Q. For what values of x is $\tan x$ undefined on the interval $0 \leq x \leq 2\pi$?

A. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

Q. Sketch the graph of $y = \tan x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

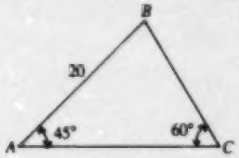
A.



Q. If $y = 2 \sin x$, what is the range of values y can have?

A. $-2 \leq y \leq 2$

52. Applications of the Law of Sines and the Law of Cosines

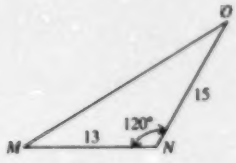


Q. Use the Law of Sines to find the length of side BC in ΔABC above.

Solution: $\frac{20}{\sin 60°} = \frac{BC}{\sin 45°}$

$\frac{20}{\frac{\sqrt{3}}{2}} = \frac{BC}{\frac{\sqrt{2}}{2}}$

A. $BC = \frac{20}{3}\sqrt{6}$



Q. Use the Law of Cosines to find the length of side MO in ΔMNO above.

Solution:

$(MO)^2 = (MN)^2 + (NO)^2 - 2(MN)(NO)(\cos N)$

$(MO)^2 = 169 + 225 - 2(13)(15)(-\frac{1}{2})$

A. $MO = \sqrt{589}$

≈ 24

COORDINATE (ANALYTIC) GEOMETRY AND FUNCTIONS

53. Midpoint Formula

Q. What are the coordinates of the midpoint of the line segment with end points (3, 5) and (−2, −7)?

A. $(\frac{1}{2}, -1)$

54. Distance Formula

Q. What is the distance between the points (−10, 5) and (10, 3)?

A. $2\sqrt{101}$

55. Lines

Q. The line through the points (1, 2) and (k, 14) has a slope of 6. What is the value of k?

A. $k = 3$

Q. Find an equation of the line containing the point (1, 4) that is perpendicular to the graph of $2x - 5y = 10$.

A. $5x + 2y = 13$

56. Conics (circles, parabolas, ellipses, hyperbolas)

Q. Match the following equations to their figures.

1. $2x^2 - 3y^2 = 10$	a. ellipse
2. $x^2 + 2x + y^2 - 3y = 7$	b. hyperbola
3. $y = 2x^2 + 11$	c. parabola
4. $x^2 + 5y^2 = 8$	d. circle

A. 1—b
2—d
3—c
4—a

Q. Determine the center and radius of the circle with equation $4x^2 + 4y^2 - 4y = 7$.

A. Center: $(0, \frac{1}{2})$

Radius: $\sqrt{2}$

Q. Determine the length of the major and minor axes of the ellipse with equation $4x^2 + 9y^2 = 36$.

A. Major axis: 6
Minor axis: 4

57. Functions—Evaluation

Q. If $f(x) = x^2$, what is $f(x-h)$?

A. $x^2 - 2xh + h^2$

Q. If $f(x) = \begin{cases} 2x + 1, \text{ for } x < 1 \\ x^2 - 3, \text{ for } x \geq 1 \end{cases}$, what is $f(1)$?

A. -2

APPENDIX E continued

58. Functions—Arithmetic and Composition

Q. Given $f(x) = 2x^2$ and $g(x) = \sqrt{1 - x}$.

a) Find a formula for $h(x)$ if $h = f \cdot g$. Give the domain of h.

A. $h(x) = 2x^2\sqrt{1 - x}$; $\{x: x \leq 1\}$

Q. b) Find an expression for $g(f(x))$. State for what values of x it is defined.

A. $g(f(x)) = \sqrt{1 - 2x^2}$; $\{x: |x| \leq \frac{1}{\sqrt{2}}\}$

59. Functions—Inverses

Q. If $f(x) = \sqrt{x}$, what is the range of $f^{-1}(x)$?

A. $\{y: y \geq 0\}$

Q. If $f(x) = \frac{1}{x - 1}$, determine $f^{-1}(3)$.

A. $\frac{4}{3}$

Q. If $f(x) = \log_b x$, then $f^{-1}(x) =$

A. b^x

60. Functions—Zeros

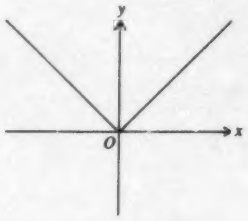
Q. Find the zeros of the polynomial function $F(x) = x^3 + 4x^2 - 4x - 16$.

A. $-4, -2, 2$

61. Graphing Absolute Value Functions

Q. Sketch the graph of $y = |x|$.

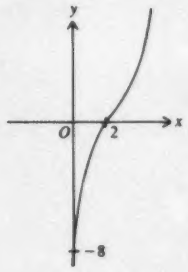
A.



62. Graphing Polynomial Functions

Q. Sketch the graph of $y = (x - 2)^3$.

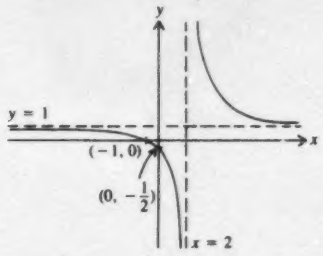
A.



63. Graphing Rational Functions

Q. Sketch the graph of $y = \frac{x + 1}{x - 2}$.

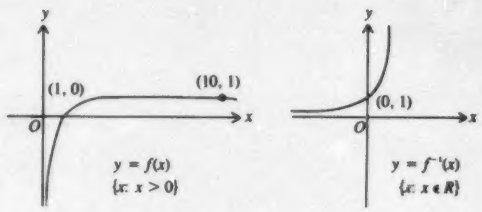
A.



64. Graphing Exponential and Logarithmic Functions

Q. Determine the domains and sketch the graphs of $f(x) = \log_{10} x$ and its inverse.

A.

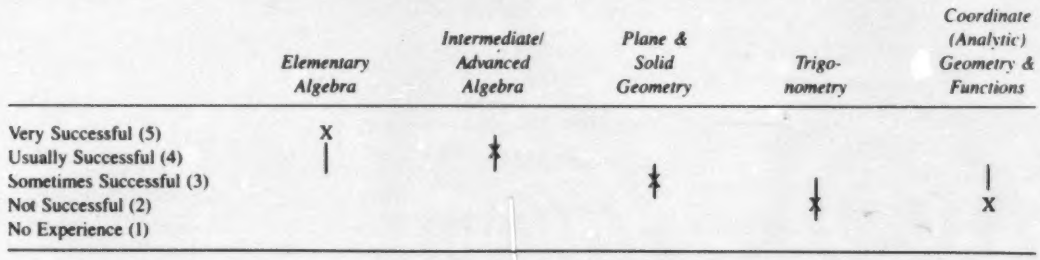


APPENDIX F. PROTOTYPE OF REPORT FOR AN INDIVIDUAL STUDENT

Profile of Preparation in Mathematics

Report for: John L. Doe
Inventory completed: Spring of twelfth grade (1983)

Five subject areas in mathematics are shown in the chart below. You have received a score for each subject area. This score, marked with an "X", reflects your level of preparation in each subject area according to your responses on the Mathematic Inventory you completed recently. For comparison, the solid line shows the *50 percent range* (that is, the middle 50 percent of the students) in each subject area for all students completing the inventory nationwide.



HOW YOUR LEVEL OF PREPARATION WAS DETERMINED

Recall that the Mathematics Inventory covered the five subject areas shown above, with several topics listed within each subject area. (Please refer to your copy of the Mathematics Inventory for examples of the topics.) For each topic, you were asked how well you could solve problems in that topic at the time you learned it. Your response to each topic was assigned a number as follows:

Very successful = 5
Usually successful = 4
Sometimes successful = 3
Not successful = 2
No experience (did not study topic) = 1

For a given subject area, your responses to all the topics in that area were combined, taking into account ratings of the importance of each topic made by a nationwide sample of secondary school mathematics teachers. These importance ratings, along with a fuller explanation of how your score was computed, are presented in the "Mathematics Profile Manual" on file with your school's guidance counselor.

You may wish to discuss your strengths and weaknesses in mathematics with your guidance counselor. If you plan to attend college, you may want to discuss the types of topics you need to have learned in order to take certain courses. To assist you in your self-evaluation, those topics in which you may need further preparation are listed below. These are the topics in which you did *not* rate yourself as "very successful" or "usually successful."

Elementary Algebra:

(a) simplifying radicals; (b) simplifying rational expressions—concepts

Intermediate/Advanced Algebra:

(a) solving logarithmic equations; (b) solving inequalities: quadratic and other advanced; (c) binomial expansion

Plane and Solid Geometry:

(a) deductive reasoning and proof

Trigonometry:

(a) proof of identities; (b) domain and range

Coordinate (Analytic) Geometry and Functions:

(a) functions—inverses; (b) graphing polynomial functions; (c) graphing exponential and logarithmic functions

You may also wish to know how you compare with other students completing the inventory nationwide with respect to command of individual topics. Below are listed the topics in which your response was either above or below the 50 percent range nationwide.

In *Elementary Algebra* your response was above that of the typical student (that is, above the 50 percent range) in: (a) operations with signed numbers; (b) factoring polynomial expressions—elementary; and (c) translations and word problems. Your response was below that of the typical student in: (a) simplifying radicals.

In *Intermediate/Advanced Algebra* your response was above that of the typical student in: (a) evaluating expressions with negative integer exponents; (b) evaluating expressions with fractional exponents; and (c) operations with rational expressions. Your response was below that of the typical student in: (a) solving logarithmic equations; and (b) binomial expansion.

In *Plane and Solid Geometry* your response was above that of the typical student in: (a) triangles—general; and (b) right triangles. Your response was below that of the typical student in no topic.

In *Trigonometry* your response was above that of the typical student in no topic. Your response was below that of the typical student in: (a) proof of identities.

In *Coordinate (Analytic) Geometry and Functions* your response was above that of the typical student in: (a) conics. Your response was below that of the typical student in: (a) graphing polynomial functions; and (b) graphing exponential and logarithmic functions.

Note that the above information is presented simply to assist you in your self-evaluation. It is ultimately your own responsibility to examine your knowledge of mathematics and compare it with what you need to have learned in order to take certain courses or otherwise achieve your educational goals.

APPENDIX G. PROTOTYPE OF REPORT FOR A SECONDARY SCHOOL

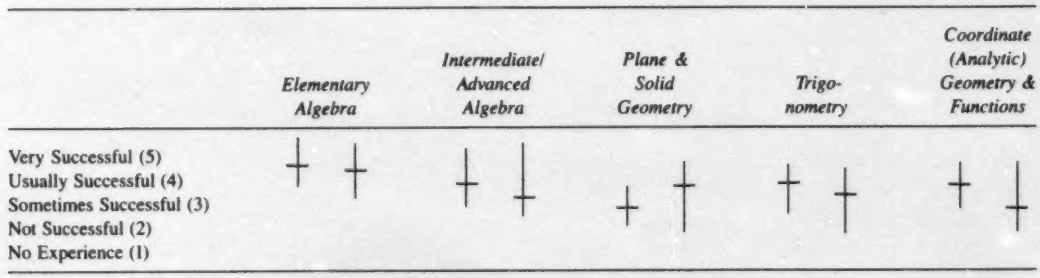
Profile of Preparation in Mathematics

Report for: Eleanor R. Jones High School
Inventory completed: Spring 1983

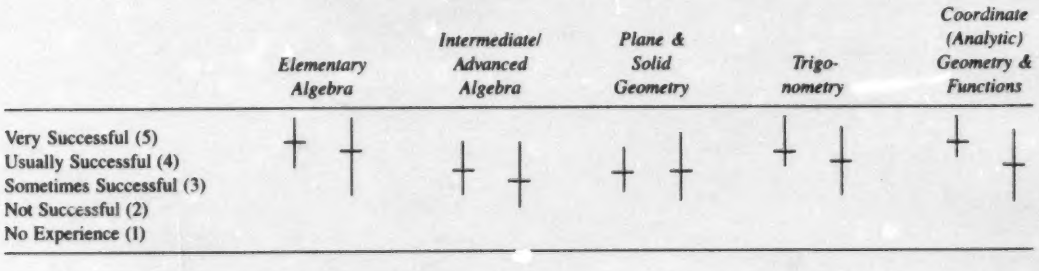
Your students recently completed the Mathematics Inventory which was designed to determine their preparation in each of five subject areas: (a) elementary algebra; (b) intermediate/advanced algebra; (c) plane and solid geometry; (d) trigonometry; and (e) coordinate (analytic) geometry and functions. The following report is an overview of the responses of students in your school. The method of computing each student's level of preparation, along with a copy of the inventory, are presented later in this report.

In each chart are two vertical lines for each of the five subject areas. The first vertical line shows the 50 percent range (middle 50 percent of students) for students in your school (shown separately for your eleventh and twelfth grade students), with the median student in your school shown by the horizontal bar intersecting the line. The second vertical line shows the 50 percent range for all students completing the inventory nationwide, with the median student nationwide shown by the horizontal bar.

Grade 11



Grade 12



HOW A STUDENT'S LEVEL OF PREPARATION WAS DETERMINED

The Mathematics Inventory covers the five subject areas shown above, with several topics listed within each subject area (see attached copy of the Mathematics Inventory). For each topic, the students were asked how well they could solve problems in that topic at the time they learned it. A student's response to a topic was assigned a number as follows:

Very successful = 5
Usually successful = 4
Sometimes successful = 3
Not successful = 2
No experience (did not study topic) = 1

For a given subject area, the students' responses to all the topics in that area were combined, taking into account teachers' ratings of the importance of each topic. These importance ratings, as determined by a nationwide survey of secondary school mathematics teachers, are presented in the Mathematics Profile Manual, along with further explanation of the method by which students' scores were computed.

It is useful also to examine the students' self-reported strengths and weaknesses within each subject area. Below are tables showing your students' self-reported proficiency for individual topics within each subject area. The students are subdivided into Grades 11 and 12; also, data are presented for those students who reported having taken a course in the subject area named in the left column (that is, elementary algebra, intermediate/advanced algebra, etc.). For each of these groups, the following information is presented for each topic: (a) the median response of all students in the specified group; (b) the percentage of students indicating in the inventory that they did not study the topic listed; and (c) the median response of the students who, according to their inventory responses, *did* study that topic. (Note that the latter responses fall on a scale from 2 to 5.) These data are provided as an aid to you in identifying areas of strength and weakness in your students' preparation.

Responses of Students in Your School for Each Topic on Mathematics Inventory

	Grade 11			*Grade 12*			*Students reporting having taken course in subject area indicated*		
	Median score of all students	*% not studying topic*	*Median score of students studying topic*	*Median score of all students*	*% not studying topic*	*Median score of students studying topic*	*Median score of all students*	*% not studying topic*	*Median score of students studying topic*
Elementary Algebra									
1 operations with signed numbers	X*	X	X	X	X	X	X	X	X
2 substitution									
3 simplifying expressions with nonnegative integer exponents									
4 simplifying polynomial expressions									
5 factoring polynomial expressions—elementary									
6 solving simple equations—1 unknown									
7 solving inequalities—linear/other elementary									
8 graphing linear equations									
9 solving systems of 2 linear equations									
10 translations and word problems									
11 simplifying radicals									
12 simplifying rational expressions—concepts									
Intermediate/Advanced Algebra									
13 evaluating expressions with negative integer exponents									
14 evaluating expressions with fractional exponents									
15 operations involving radicals									
16 factoring polynomial expressions—advanced									
17 finding polynomial equations									
18 operations with rational expressions—advanced									
19 operations with complex numbers									

*Xs represent numbers that appear under each entry for items 1–64.

Responses of Students in Your School for Each Topic on Mathematics Inventory—Continued

	Grade 11			*Grade 12*			*Students reporting having taken course in subject area indicated*		
	Median score of all students	*% not studying topic*	*Median score of students studying topic*	*Median score of all students*	*% not studying topic*	*Median score of students studying topic*	*Median score of all students*	*% not studying topic*	*Median score of students studying topic*
20 logarithms—concepts	x[a]	x	x	x	x	x	x	x	x
21 solving fractional equations									
22 solving simple radical equations									
23 solving exponential equations									
24 solving logarithmic equations									
25 solving literal equations									
26 solving quadratic equations									
27 solving inequalities: quadratic/other advanced									
28 graphing linear equations; finding slope and intercepts									
29 graphing systems of linear inequalities									
30 graphing quadratic equations and inequalities									
31 solving systems of equations—advanced									
32 binomial expansion									
33 sequences and series									
Plane and Solid Geometry									
34 parallel lines									
35 triangles—general									
36 right triangles									
37 applications of the Pythagorean Theorem									
38 interior angles of polygons									
39 circles and inscribed figures									
40 deductive reasoning and proof									
41 rectangular solids									
42 right circular cylinders and cones									
43 spheres									
Trigonometry									
44 angle measure									
45 triangles/angles									
46 equations									
47 knowledge of identities									
48 proof of identities									
49 inverses									
50 graphs									
51 domain and range									
52 application of the Law of Sines and the Law of Cosines									
Coordinate (Analytic) Geometry and Functions									
53 midpoint formula									
54 distance formula									
55 lines									
56 conics (circles, parabolas, ellipses, hyperbolas)									
57 functions—evaluation									
58 functions—arithmetic and composition									
59 functions—inverses									
60 functions—zeros									
61 graphing absolute value functions									
62 graphing polynomial functions									
63 graphing rational functions									
64 graphing exponential and logarithmic functions									

[a]Xs represent numbers that appear under each entry for items 1–64.